THE MATHEMATICS TEACHER

VOLUME XIV MARCH, 1921 NUMBER 3

# TERMS AND SYMBOLS IN ELEMENTARY MATHEMATICS.

A PRELIMINARY REPORT BY

THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.*

**A. Limitations Imposed by the Committee upon its Work.**— The Committee feels that it should place upon itself certain general limitations, as follows:

1. No attempt should be made to impose the phraseology of any definition upon the schools, although the Committee should state clearly its general views as to the meaning of disputed terms.

2. No effort should be made to change any well-defined current usage unless there is a strong reason for doing so, which reason is supported by the best authority.

3. So far as reasonable, *the terms used should be international.* This principle excludes the use of all individual efforts at coining new terms except under circumstances of great urgency. The individual opinions of the members, as indeed of any teacher or body of teachers, should have little weight in comparison with international usage if this usage is definite. If an idea has to be expressed so often in elementary mathematics that it becomes necessary to invent a single term or symbol for the purpose, this invention is necessarily the work of an individual; but it is highly desirable, even in this case,

* The first draft of this report was prepared by a subcommittee consisting of David Eugene Smith (chairman), W. W. Hart, H. E. Hawkes, E. R. Hedrick and H. E. Slaught. It was revised by the National Committee at its meeting December 29 and 30, 1920, and is now issued as one of its preliminary reports.

that it should receive the sanction of high authority before it is used in any system of examinations.

4. On account of the large number of terms and symbols now in use, the recommendations to be made should be typical rather than exhaustive.

## I. Geometry.

**B. Undefined Terms.**—The Committee recommends that little attempt be made to define, with any approach to precision, terms whose definitions are not needed as parts of a proof.

Especially is it recommended, that no attempt be made to define completely such terms as *space*, *magnitude*, *point*, *straight line*, *surface*, *plane*, *direction*, *distance*, and *solid*, although the significance of such terms should be made clear by informal explanations and discussions.

**C. Definite Usuage Recommended.**—While not seeking to dictate the phraseology of any definition, it is the opinion of the Committee that the following general usage is desirable:

1. *Circle* should be considered as the curve; but where no ambiguity arises, the word "circle" may be used to refer either to the curve or to the part of the plane inclosed by it.

2. *Polygon* (including *triangle*, *square*, *parallelogram*, and the like) should be considered, by analogy to a circle, as a closed broken line; but where no ambiguity arises, the word polygon may be used to refer either to the broken line or to the part of the plane inclosed by it.

3. *Area of a circle* should be defined as the area (numerical measure) of the portion of the plane inclosed by the circle. Similarly, *segment of a circle* should be defined as that portion of the plane bounded by a chord and its arc. *Area of a polygon* should be treated in the same way.

4. *Solids.* The usage above recommended with respect to plane figures is also recommended with respect to solids. For example, *sphere* should be regarded as a surface, its volume should be defined in a manner similar to the area of a circle, and the double use of the word should be allowed where no ambiguity arises. A similar usage should obtain with respect to such terms as *polyhedron*, *cone*, and *cylinder*.

5. *Circumference* should be considered as the length

(numerical measure) of the circle (line). Similarly, *perimeter* should be defined as the length of the broken line which forms a polygon, that is, as the sum of the lengths of the sides.

6. *Obtuse angle* should be defined as an angle greater than a right angle and less than a straight angle, and should therefore not be defined merely as an angle greater than a right angle.

7. The term *right triangle* should be preferred to "right-angled triangle," this usage being now so standardized in this country that it may properly be continued in spite of the fact that it is not international. Similarly for *acute triangle*, *obtuse triangle*, and *oblique triangle*.

8. Such English plurals as *formulas*, and *polyhedrons*, should be used in place of the Latin and Greek plurals. Such unnecessary Latin abbreviations as *Q.E.D.* and *Q.E.F.* should be dropped.

9. The definitions of *axiom* and *postulate* vary so much that the Committee does not undertake to distinguish between them.

**D. Terms Made General.**—It is the recommendation of the Committee that the modern tendency of having terms made as general as possible should be followed. For example:

1. *Isosceles triangle* should be defined as a triangle having two equal sides. There should be no limitation to two and only two equal sides.

2. *Rectangle* should be considered as including a square as a special case.

3. *Parallelogram* should be considered as including a rectangle, and hence a square, as a special case.

4. *Segment* should be used to designate the part of a straight line included between two of its points as well as the part of the plane bounded by an arc of a circle and its chord, this being the usage generally recognized by modern writers.

**E. Terms to Be Abandoned.**—It is the opinion of the Committee that the following terms are not of enough consequence in elementary mathematics at the present time to make their recognition desirable in examinations and that they serve chiefly to increase the technical vocabulary to the point of being burdensome and unnecessary:

1. *Antecedent* and *consequent*.

2. *Third proportional* and *fourth proportional*.

3. *Equivalent.* An unnecessary substitute for the more precise expressions "equal in area" and "equal in volume," or (where no confusion is likely to arise) for the single word "equal."

4. *Trapezium.*

5. *Scholium*, *lemma*, *oblong*, *scalene triangle*, *sect*, *perigon*, *rhomboid* (the term "oblique parallelogram" being sufficient), and *reflex angle* (in elementary geometry).

6. Terms like *flat angle*, *whole angle*, and *conjugate angle* are not of enough value in an elementary course to make it desirable to recommend them.

7. *Subtend*, a word which has no longer any etymological meaning to most students and teachers of geometry. While its use will naturally continue for some time to come, teachers may safely incline to such forms as the following: "In the same circle equal arcs *have* equal chords."

8. *Homologous*, the less technical term "corresponding" being preferable.

9. Guided by principle A3 and its interpretation, the Committee advises against the use of such terms as the following: *Angle-bisector*, *angle-sum*, *consecutive interior angles*, *supplementary consecutive-exterior angles*, *quader* (for rectangular solid), *sect*, *explement*, *transverse angles.*

10. It is unfortunate that it still seems to be necessary to use such a term as *parallelepiped*, but we seem to have no satisfactory substitute. For rectangular parallelepiped, however, the use of *rectangular solid* is recommended. If the terms were more generally used in elementary geometry it would be desirable to consider carefully whether better ones could not be found for the purposes than *isoperimetric*, *apothem*, *icosahedron*, and *dodecahedron.*

**F. Symbols in Elementary Geometry.**—It should be recognized that a symbol like $\perp$ is merely a piece of shorthand designed to abbreviate and to allow for an easy grasp of a written or printed statement. Many teachers and a few writers make an extreme use of symbols, coining new ones to meet their own views as to usefulness, and this practice is naturally open to

objection.* There are, however, certain symbols that are international, and certain others of which the meaning is at once apparent and which are sufficiently useful and generally enough recognized to be recommended.

For example, the symbols for triangle, $\triangle$, and circle, $\odot$, are international, although used more extensively in the United States than in other countries. Their use, with their customary plurals is recommended.

The symbol $\perp$, generally read as representing the single word "perpendicular" but sometimes as standing for the phrase "is perpendicular to," is fairly international and the meaning is apparent. Its use is therefore recommended. On account of such a phrase as "the $\perp AB$," the first of the above readings is likely to be the more widely used, but in either case there is no chance for confusion.

The symbol $\|$ for "parallel" or "is parallel to" is fairly international and is recommended.

The symbol $\sim$ for "similer" or "is similar to" is international and is recommended.

The symbols $\simeq$ and $\equiv$ for "congruent" or "is congruent to" both have a considerable use in this country. The Committee feels that the former, which is fairly international, is to be preferred because it is the more distinctive and suggestive.

The symbol $\angle$ for angle is, because of its simplicity, coming to be generally preferred to any other and is therefore recommended.

Since the following terms are not used frequently enough to render special symbols of any particular value, the world has not developed any that have general acceptance, and there seems to be no necessity for making the attempt: Square, rectangle, parallelogram, trapezoid, quadrilateral, semicircle.

The symbol $\overset{\frown}{AB}$ for arc $AB$ cannot be called international. While the value of the symbol $\frown$ in place of the short word *arc* is doubtful, the Committee sees no objection to its use.

The symbol $\therefore$ (therefore) has a value that is generally

* This is not intended to discourage the use of algebraic methods in the solution of such geometric problems as lend themselves readily to algebraic treatment.

recognized, but the symbol $\therefore$ (since) is used so seldom that it should be abandoned.

With respect to the lettering of figures, the Committee calls attention for purposes of general information, to a convenient method, found in certain European and in some American textbooks, of lettering triangles: Capitals represent the vertices, corresponding small letters represent opposite sides, corresponding small Greek letters represent angles, and the primed letters represent the corresponding parts of a congruent triangle. This permits of speaking of $\alpha$ (alpha) instead of "angle $A$," and of "small $a$" instead of $BC$. The plan is by no means international with respect to the Greek letters. The Committee is prepared, however, to recommend it with the optional use of the Greek forms.

In general, it is recommended that a single letter be used to designate any geometrical magnitude, whenever there is no danger of ambiguity. The use of numbers to designate angles, such as $\angle 1$, $\angle 2$, etc., should, however, be abandoned.

With respect to the symbolism for limits, the Committee calls attention to the fact that the symbol $\doteq$ is a local one, the symbol $\rightarrow$ (for "tends to") being both international and expressive, and having constantly grown in favor in recent years. Although the subject of limits is not generally treated scientifically in the secondary school, the idea is mentioned in geometry and a symbol may occasionally be needed.

While the teacher should be allowed freedom in the matter, the Committee feels that it is desirable to discourage the use of such purely local symbols as the following:

$\doteq^{\circ}$ for "equal in degrees,"

*ass* for "two sides and an angle adjacent to one of them," and

*sas* for "two sides and the included angle."

**G. Terms not Standardized.**—At the present time, there is not sufficient agreement upon which to base recommendations as to the use of the term "ray" and as to the value of terms like "coplanar," "collinear" and "concurrent" in elementary work. Many terms, similar to these, will gradually become standardized or else will naturally drop out of use.

## II. Algebra and Arithmetic.

**H. Terms in Algebra.**—1. With respect to equations the Committee calls attention to the fact that the classification according to degree is comparatively recent and that this probably accounts for the fact that the terminology is so unsettled. The Anglo-American custom of designating an equation of the first degree as a *simple equation* has never been satisfactory, because the term has no real significance. The most nearly international terms are *equation of the first degree* (or "first degree equation") and *linear equation*. The latter is so brief and suggestive that it should be generally adopted.

2. The term *quadratic equation* (for which the longer term "second degree equation" is an unnecessary synonym, although occasionally a convenient one) is well established. The terms *pure quadratic* and *affected quadratic* signify nothing to the pupil except as he learns the meaning from a book and the Committee recommends that they be dropped. Terms more nearly in general use are *complete quadratic* and *incomplete quadratic*. The Committee feels, however, that the distinction thus denoted is not of much importance and believes that it can well be dispensed with in elementary instruction.

3. As to other special terms, the Committee recommends abandoning, so far as possible, the use of the following: *aggregation* for grouping; *vinculum* for bar; *evolution* for finding roots, as a general topic; *involution* for finding powers; *extract* for find (a root); *absolute term* for constant term; *multiply an equation*, *clear of fractions*, *cancel* and *transpose*, at least until the significance of the terms is entirely clear; *aliquot part* (except in commercial work).

4. The Committee also advises the use of either *system of equations* or *set of equations* instead of "simultaneous equations," in such an expression as "solve the following set of equations," in view of the fact that at present no well established definite meaning attaches to the term "simultaneous."

5. The term *simplify* should not be used in cases where there is possibility of misunderstanding. For purposes of computation, for example, the form $\sqrt{8}$ may be simpler than the form $2\sqrt{2}$, and in some cases it may be better to express $\sqrt{\frac{3}{4}}$ as $\sqrt{0.75}$ instead of $\frac{1}{2}\sqrt{3}$. In such cases, it is better to give more

explicit instructions than to use the misleading term "simplify."

6. The Committee regrets the general uncertainty in the use of the word *surd*, but it sees no reasonable chance at present of replacing it by a more definite term. It recognizes the difficulty generally met by young pupils in distinguishing between *coefficient* and *exponent*, but it feels that it is undesirable to attempt to change terms which have come to have a standardized meaning and which are reasonably simple. These considerations will probably lead to the retention of such terms as *rationalize*, *extraneous root*, *characteristic* and *mantissa*, although in the case of the last two terms, "integral part" and "fractional part" (of a logarithm) would seem to be desirable substitutes.

7. While recognizing the motive that has prompted a few teachers to speak of "positive $x$" instead of "plus $x$," and "negative $y$" instead of "minus $y$," the Committee feels that attempts to change general usage should not be made when based upon trivial grounds and when not sanctioned by mathematicians generally.

**I. Symbols in Algebra.**—The symbols in elementary algebra are now so well standardized as to require but few comments in a report of this kind. The Committee feels that it is desirable, however, to call attention to the following details:

1. Owing to the frequent use of the letter $x$, it is preferable to use the center dot (a raised period) for multiplication in the few cases in which any symbol is necessary. For example, in a case like $1 \cdot 2 \cdot 3 \cdots (x-1) \cdot x$, the center dot is preferable to the symbol $\times$; but in cases like $2a(x-a)$ no symbol is necessary. The Committee recognizes that the period (as in $a.b$) is more nearly international than the center dot (as in $a \cdot b$); but inasmuch as the period will continue to be used in this country as a decimal point, it is likely to cause confusion, to elementary pupils at least, to attempt to use it as a symbol for multiplication.

2. With respect to division, the symbol $\div$ is purely Anglo-American, the symbol : serving in most countries for division as well as ratio. Since neither symbol plays any part in business life, it seems proper to consider only the needs of algebra,

and to make more use of the fractional form and (where the meaning is clear ) of the symbol /, and to drop the symbol ÷ in writing algebraic expressions.

3. With respect to the distinction between the use of + and — as symbols of operation and as symbols of direction, the Committee sees no reason for attempting to use smaller signs for the latter purpose, such an attempt never having received international recognition, and the need of two sets of symbols not being sufficient to warrant violating international usage and to make it necessary to burden the pupil with this additional symbolism.

4. With respect to the distinction between the symbols $\equiv$ and $=$ as representing respectively identity and equality, the Committee calls attention to the fact that, while the distinction is generally recognized, the consistent use of the symbols is rarely seen in practice. The Committee recommends that the symbol $\equiv$ be not employed in examinations for the purpose of indicating identity. The teacher, however, should use both symbols if desired.

5. With respect to the root sign, $\sqrt{\ }$, the Committee recognizes that convenience of writing assures its continued use in many cases instead of the fractional exponent. It is recommended, however, that in algebraic work involving complicated cases the fractional exponent be preferred. Attention is also called to the fact that the convention is quite generally accepted that the symbol $\sqrt{a}$ ($a$ representing a positive number) means only the positive square root and that the symbol $\sqrt[n]{a}$ means only the principal $n$th root, and similarly for $a^{1/2}$, $a^{1/n}$. The reason for this convention is apparent when we come to consider the value of $\sqrt{4}+\sqrt{9}+\sqrt{16}+\sqrt{25}$. This convention being agreed to, it is improper to write $x=\sqrt{4}$, as the complete solution of $x^2-4=0$, but the result should appear as $x=\pm\sqrt{4}$. Similarly, it is not in accord with the convention to write $\sqrt{4}=\pm 2$, the conventional form being $\pm\sqrt{4}=\pm 2$; and for the same reason it is impossible to have $\sqrt{(-1)^2}=-1$, since the symbol refers only to a positive root. These distinctions are not matters to be settled by the individual opinion of a teacher or a local group of teachers; they are purely matters of convention and the Committee simply sets forth, for the

benefit of the teachers, this statement as to what the convention seems to be among the leading writers of the world at the present time.

When imaginaries are used, the symbol $i$ should be employed instead of $\sqrt{-1}$ except possibly in the first presentation of the subject.

7. As to the factorial symbols $5!$ and $\lfloor 5$, to represent $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, the tendency is very general to abandon the second one, probably on account of the difficulty of printing it, and the committee so recommends. This question is not, however, of much importance in the general courses in the high school.

8. With respect to symbols for an unknown quantity there has been a noteworthy change within a few years. While the Cartesian use of $x$ and $y$ will doubtless continue for two general unknowns, the recognition that the formula is, in the broad use of the term, the central feature of algebra has led to the extended use of the initial letter. This is simply illustrated in the direction to solve for $r$ the equation $A = \pi r^2$. This custom is now international and should be fully recognized in the schools.

9. The Committee advises abandoning the double colon ( : : ) in proportion, and the symbol $\propto$ in variation, both of these symbols being now practically obsolete.

**J. Terms and Symbols in Arithmetic.**—1. While it is rarely wise to attempt to abandon suddenly the use of words that are well established in our language, the Committee feels called upon to express regret that we still require very young pupils, often in the primary grades, to use such terms as *subtrahend*, *addend*, *minuend* and *multiplicand*. Teachers themselves rarely understand the real significance of these words, nor do they recognize that they are comparatively modern additions to what used to be a much simpler vocabularly in arithmetic. The Committee recommends that such terms be used, if at all, only after the sixth grade.

2. Owing to the uncertainty attached to such expressions as "to three decimal places," "to thousandths," "correct to three decimal places," "correct to the nearest thousandth," the following usage is recommended: When used to specify accuracy in computation, the four expressions should be regarded as identical. The expression "to three decimal places" or "to

thousandths" may be used in giving directions as to the extent of a computation. It then refers to a result carried only to thousandths, without considering the figure of ten-thousandths; but it should be avoided as far as possible because it is open to misunderstanding. As to the term, "significant figure," it should be noted that 0 is always significant except when used before a decimal fraction to indicate the absence of integers or, in general, when used merely to locate the decimal point. For example, the zeros underscored in the following are "significant," while the others are not: 0.5, 9.50, 102, 30,200. Further, the number 2396, if expressed correct to three significant figures, would be written 2400. It should be noted that the context or the way in which a number has been obtained is sometimes the determining factor as to the significance of a 0.

3. The pupil in arithmetic needs to see the work in the form in which he will use it in practical life outside the schoolroom. His visualization of the process should therefore not include such symbols as $+$, $-$, $\times$, $\div$, which are helpful only in writing out the analysis of a problem or in the printed statement of the operation to be performed. Because of these facts the Committee recommends that only slight use be made of these symbols in the written work of the pupil, except in the analysis of problems. It recognizes, however, the value of such symbols in printed directions and in these analyses.

### III. General Observations and Recommendations.

**K. General Observations.**—The Committee desires also to record its belief in two or three general observations.

1. It is very desirable to bring mathematical writing into closer touch with good usage in English writing in general. That we have failed in this particular has been the subject of frequent comment by teachers of mathematics as well as by teachers of English. This is all the more unfortunate because mathematics may be and should be a genuine help towards the acquisition of good habits in the speaking and writing of English. Under present conditions, with a style that is often stilted and in which undue compression is evident, we do not offer to the student the good models of English writing of which mathematics is capable nor, indeed, do we always offer good models of thought processes. It is to be feared that many

teachers encourage the use of a kind of vulgar mathematical slang when they allow such words as "tan" and "cos," for tangent and cosine, and habitually call their subject by the title "math."

2. In the same general spirit the Committee wishes to observe that teachers of mathematics and writers of textbooks often seem to have gone to an extreme in searching for technical terms and for new symbols. The Committee expresses the hope that mathematics may retain as far as possible a literary flavor. It seems perfectly feasible that a printed discussion should strike the pupil as an expression of reasonable ideas in terms of reasonable English forms of expression. The fewer technical terms we introduce, the less is the subject likely to give the impression of being difficult and a mere juggling of words and symbols.

3. While recognizing the claims of euphony, the fact that a word like "historic" has a different meaning from "historical" and that confusion may occasionally arise if "arithmetic" is used as an adjective with a different pronounciation from the noun, the Committee advises that such forms as geometric be preferred to geometrical. This is already done in such terms as analytic geometry and elliptic functions, and it seems proper to extend the custom to include *arithmetic*, *geometric*, *graphic* and the like.

**L. General Recommendations**—In view of the desirability of a simplification of terms used in elementary instruction, and of establishing international usage so far as reasonable, the Committee recommends that the subject of this report be considered by a committee to be appointed by Section IV of the next International Congress of Mathematicians, such committee to contain representatives of at least the recognized international languages admitted to the meetings.

2. The Committee suggests that examining bodies, contributors to mathematical journals and authors of textbooks endeavor to follow the general principles formulated in this report.

Criticisms, comments and suggestions for the revision of this preliminary report are invited. They should be sent to J. W. Young, chairman, Hanover, New Hampshire.

## THE RECITATION IN MATHEMATICS.

The purpose of education is to help the individual better to meet both present and future life situations. The process by which this purpose is realized continues throughout life and the means of education includes every phase of our environment which tends to modify the way in which we react to a given situation. Clearly the school is only one of the means of education. Within the school various forms of student activity are rightfully assuming an ever increasing importance, but at present most teachers look upon the recitation as the most important educative factor in the school. The function of the recitation seems to be to pass on to the next generation an accumulation of experience which school authorities believe to be essential to the welfare of society and in so doing to develop certain desirable abilities and capacities in the individual. Unfortunately in our attempt to realize this function we have separated our subject matter from its useful relationships and in the child's mind it is a mass of material almost wholly unrelated to the world in which he lives, and our very attempts have defeated our purpose. This paper discusses certain fundamental principles which will compel a closer relationship between mathematics and out-of-school life and will therefore make the recitation a more effective means of realizing the aim of education.

*Nothing is of educational value which does not result in some form of activity on the part of the one being educated.* Not only physical but also mental and emotional activities are here included. This principle denies the value of a recitation in which the pupils are passively receptive and the teacher is the active agent. We may explain to the child how to ride a bicycle but it is only by trying that he learns. The teacher may develop mathematical theorems, solve problems, and explain processes all to little or no purpose as long as his class remains passive. Ready made proofs of our geometry texts require little or no reasoning on the part of children and there-

fore, of themselves, cannot possibly develop the power to reason. This principle demands that the pupil as well as the teacher shall be actively engaged in the work of the recitation. This does not mean that the teacher is to be eliminated. He is to stimulate and guide the activities of the pupils, but if the recitation is to be effective, they must do the work.

"*Practical skill, modes of effective technique, can be intelligently, non-mechanically used, only when intelligence has played a part in their acquisition.*"* It has been said that all the geometry that is needed for practical purposes can be taught in a very few lessons. The authors of such statements undoubtedly mean that the facts which find frequent applications in the lives of most individuals can be taught in a mechanical rule-of-thumb way in a few lessons. Such teaching, however, is a violation of the principle under discussion. If facts are thus taught without an intelligent understanding of the underlying principles, the pupil may be able to use them under the exact conditions under which they were learned, but when application is to be made to a new situation it is doubtful if he can succeed. By mere imitation we can, in a very short time, teach a pupil to bisect a line by the usual method of constructing two intersecting arcs on either side of the line and connecting the two intersections. The chances are that he will then fail to bisect a given length laid off along the intersection of the floor and wall. If, however, he has been taught that *just any two points* equidistant from the extremities of the line determine the perpendicular bisector of the line, the chances of his solving the situation are much greater. In the problem $3\overline{)948}$, the child is taught in a mechanical way that the remainder 1 obtained from dividing 4 by 3 together with 8 make 18. Later, even in high school, in the problem $3\overline{)964\frac{3}{8}}$ he assumes that the remainder 1 and 3 make 13, and he consequently divides $1\frac{3}{8}$ instead of $1\frac{1}{8}$ by 3. If, in the first case, he had been taught that he was reducing from one denomination to another, he would, in the second case, more probably have reduced the 1 to eights. Examples can be multiplied, but it is clear that if mathematical education is to be most useful, there must be an intelligent mastery of facts. The rule-of-thumb method must be replaced by a rational method.

* Dewey, John, "How We Think," p. 52.

*Those and only those processes which will occur frequently in the lives of all should, through drill, be reduced to habit.* There is a tendency on the part of some educators to eliminate all drill from the recitation. On the other hand, there are many, especially among mathematics teachers, who devote a majority of the time to drill. The principle safeguards against both of these extremes. It at once provides for drill on fundamentals and demands that all drill on non-essential and on essential material which is only occasionally useful shall be eliminated. Efficiency demands that the number combinations, the laws of signs, the laws of exponents, and the association of certain properties with the parallelograms shall become a matter of habit. On the other hand in algebra pupils are drilled on long division, forms of factoring, difficult fractions, and complicated fractional equations none of which can by any chance be of value to the majority of pupils. In trigonometry there are difficult formulæ which are necessary but which will occur only occasionally in the lives of most people. Pupils are compelled to memorize them although they forget them immediately after examinations. Instead of memorizing such material, the time should be spent on learning how to use it. When the need arises the pupil can look it up, which he will have to do any way as he will have long since forgotten it. Just as surely as efficiency demands that certain processes shall be reduced to habit, it also demands that we shall not drill on material which will not occur frequently in the lives of nearly all people.

*Only those things which have been learned in useful relationships will surely be useful.* The fact that a pupil has learned certain phases of pure mathematics does not mean that he will recognize its usefulness in the solution of problems outside of the class-room. Students who can solve quadratic equations in the algebra class fail to use them in the physics class. Only recently a student who had completed the study of plane and solid geometry, failed completely to recognize a geometrical situation in the question, "How can a sailor determine how many degrees of latitude he is north of the equator?" A student of calculus who could easily find the value of $x$ which gives a maximum for a function of $x$, never so much

as associated calculus with the problem, "How large must be the squares cut from the corners of a rectangular piece of cardboard 10 inches by 20 inches so that a box of the largest possible capacity may be made? If, however, we should begin with practical problems and let the formal phases of mathematics grow out of them, and if we should follow this up with a variety of applications and encourage our pupils to look for other applications, mathematics will become a useful tool for the solution of practical problems whenever they are met.

Dr. Thorndike has said,* "Teach nothing merely because of its disciplinary value, but teach everything so as to get what disciplinary value it does have." *The best way to get what disciplinary value mathematics has is to get the greatest possible practical value it has.*† Perhaps geometry has been defended more strongly as a means of developing the ability to think than any other subject. However, if we examine the text-book method by which geometry is usually taught we find that almost no element of the thought process is present. Dewey‡ analyzes the act of thought into the following five steps, "(i) a felt difficulty; (ii) its location and definition; (iii) suggestion of possible solution; (iv) development by reasoning of bearings of the suggestion; (v) further observation and experiment leading to its acceptance or rejection; that is the conclusion of belief or disbelief." Ordinarily no one of these steps is present in the text-book presentation of geometry. The student does not feel a real purpose in proving a theorem; he does not define the problem or state what is to be done. Neither does he propose and develop a possible solution nor does he draw a conclusion. In fact all of these are done for him and he too frequently memorizes the text and *recites* it to the teacher. If the thought process is not present, then surely the ability to think can not be developed.

However, let us undertake to lay off a rectangular field for a Maypole dance. After the pupils have established one side (i) they have the problem of turning a square corner; (ii) they define it as erecting a perpendicular to a line at a given point in the line; (iii) they suggest various methods of pro-

* Thorndike, E. L., "Principles of Education," p. 249.

† Mair, David, "School Course in Mathematics," p. iii.

‡ Dewey, John, "How We Think," pp. 68–72.

cedure; (iv) they consider each of these in the light of every available fact; and (v) those suggestions that give promise are carefully investigated, proved or disproved, and finally accepted or rejected. Such a process involves a complete act of thought and therefore will develop in the fullest possible way the ability to think; and it has the additional advantage of teaching the pupil to use his geometry outside the school room.

It is not possible to give a complete discussion of the mathematics recitation within the limits of this brief paper. However, we believe that the five fundamental principles stated above provide for almost all valuable phases of the recitation. If they are considered in the conduct of the recitation, there must be a conscious purpose on the part of the pupil; facts must be related to life outside the school room; we must provide for the mastery of important mathematical truths; drill on fundamentals is essential; student activity calling for co-operation within the class-room and in field work provides an opportunity for the socialized recitation; and finally useful activity requires the problem method of teaching.

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## CERTAIN MATHEMATICAL IDEALS OF THE JUNIOR HIGH SCHOOL.

The possibility of a new and stimulating course in mathematics in the Junior High School is the most encouraging feature of our recent advance in the teaching of the subject. Such a course enables the schools to break away from the present initial course and prepare the pupil for the best that mathematics has to offer in the high-school period. It enables them to take the best that other countries have to offer and to introduce it by the natural steps that psychology shows that the youth is ready to take. It allows mathematics to relate itself to the interests and apparent needs of young people instead of being presented as a purely abstract science, since the early stages of a rational course can easily be made much more apparently practical than is the case with the present course. The two standard courses in algebra and geometry, as they have come down to us, are satisfactory so far as technique and logic are concerned, and we shall make a great mistake if we lose the best features which they possess. They are not, however, good introductory courses, and we shall do very much better if we can first establish a taste for mathematics, showing numerous applications of the science and various recreations that come along with them, and then offer a set of strong electives in the Senior High School for those who show an aptitude for the subject.

The ideal plan is to give the pupil the best that can be given in each of the school years, so that, if he leaves at the end of any given year, he will have the best all-round view of the subject, up to that point, that we are able to offer. For this reason it is a great error to try to give him only arithmetic in the seventh grade, because this simply reviews what he has already learned in the way of computation. It is much better to give him a broader and more interesting view than this, particularly if in this plan we can include a continued use of arithmetic processes. Similarly, it is a mistake to try to bring

the traditional algebra down into the eighth school year, devoting the whole year to this subject. Such a plan will not give that which is best for the pupil if he leaves at the end of the year. Similarly, it is a mistake to put the business arithmetic all in the ninth year, because, although the pupil has greater maturity for appreciating its significance, if he leaves at the end of the eighth year, as a large number of pupils do, he misses this important subject.

The ideal arrangement would, therefore, seem to be that in the seventh year the pupil shall look at arithmetic from a new angle, considering it with respect to such interests as the home, the store, the most common industries, the simplest features of depositing and withdrawing money at a bank, and the use of computation in connection with mensuration. In this year he should also take up intuitive geometry, which includes the study of the common forms which everyone needs to know, the drawing of such geometric figures as people commonly need to draw, the measurement of such figures, and the locating of points on a map or on some other kind of drawing. In connection with the rules for measuring he will naturally need the formula, and thus he comes to know something of the useful shorthand of algebra and to recognize that it is a practical addition to his mental equipment. If he leaves school at the end of this year, he has now the best that we can give him from the rich field of mathematics, up to the time of his departure. He knows the most common applications of arithmetic, he knows something of graphs, he knows the common geometric forms, he has learned a little algebra in the most natural way possible, and he can use this algebra in his work in measuring simple figures.

In the eighth school year the ideal plan seems to be to carry on the algebra which the pupil has begun, to find the large and practical uses of the subject, and to apply it, where this is reasonable and convenient, to the solving of business problems in arithmetic and to simple measurements. As already said, it is very desirable to put as much arithmetic as possible in this eighth year, so as to meet the needs of that large number of pupils who will leave school at the end of this period. The work of the year may, therefore, well be chiefly the algebra of

the formula, the graph, the negative number, and the simple equation. The arithmetic may well consider those further topics which the average citizen needs to know, neglecting such complicated problems in interest and other topics as he will rarely if ever meet. With such a course we shall again give to the pupil the best that we are able to give, up to the close of the year.

The ninth school year may properly give the pupil a glimpse at a more scientific type of mathematics, for the purpose of trying his powers and tastes, and of letting him see the meaning of that type of mathematics which does large things in the civilization of today. He cannot go far in the field that will be opened before him, but it is our duty to open the gateway and give a glimpse even if we cannot give a vision. The ideal plan seems to be to give to the pupil enough work in the science of algebra to allow him to see what the subject means. This will not be a course in difficult algebraic computation, but a course which will throw light upon the work in arithmetic and will give a range of interesting exercises, illuminating as to the significance of the science. Such a course should take about fifteen weeks. Contrary to earlier practice, trigonometry follows algebra and intuitive geometry more naturally than it follows demonstrative geometry. It may be presented simply, it is interesting to pupils, and it opens an entirely new vista,—that in which the pupil sees the meaning of indirect measurement, the science by the aid of which the engineer calculates distances, the navigator finds his position at sea, and the astronomer explores the heavens. The pupil will not solve such problems as these, but he will solve numerous practical problems of a simpler nature, using the methods upon which the solution of the greater ones rests. Such a glimpse at the meaning of trigonometry will take about a month. The pupil may not remember the details for any considerable length of time, but he will remember the significance of this important branch of mathematical science. The last part of the year, say about ten or fifteen weeks, may properly be given to an introduction to the meaning of a demonstration in geometry. It is not a question of how many propositions the pupil proves, but of having him appreciate the significance of really demonstrating something, of "standing upon the vantage

ground of truth." Many a boy has found his first awakening in mathematics when and only when he has come to the stage of proving a geometric truth, and it is the duty of the schools to give to every pupil this privilege. If the pupil now finds that his mind is not adapted to mathematics, he may properly be advised to try other lines of work. But just as it is our duty to show to every pupil the beauties of good literature, of good music, and of good art, and just as it is his right and privilege to know the great events of history and the great discoveries of science, so it is our duty to show him the significance of elementary mathematics, and it is his right and privilege to know the uses and the beauties of the leading branches of the science.

The attaining of these ideals involves, however, a great responsibility. We shall not reach the goal by any unscientific, scrappy, thoughtless mixing of a little here and a little there taken at random from the conventionally treated parts of inherited mathematics. We shall not reach it by trying to "fuse" demonstrative geometry with the little-related topic of algebra, or by trying to force applications into our demonstrative work when no applications really exist. Such efforts bring all our work into disrepute. We study demonstrative geometry largely for the purpose of knowing what a demonstration means, not for the purpose of learning any considerable number of new facts. It is in the field of intuitive geometry, psychologically reached at a considerably earlier period, that we find the wide range of real applications.

Let us, therefore, attack this great problem with the determination to base our offering upon the teachings of psychology and world experience, with the idea of making our work interesting to the pupil because of its honest applications and its recreational features, and with the knowledge that the rest of the world has made this work a success and that we can do the same. If we do so, there is no reason why, in the course of a few years, we should not have elective courses in the Senior High School that are far ahead of anything that we have at present and of anything that most of us now believe to be possible.

DAVID EUGENE SMITH.

TEACHERS COLLEGE.

## GEOMETRY DETECTED BY SHERLOCK HOLMES.

A few years ago I had a class of bad boys. One day in study period I found that the head gangster—one Tom—was reading a book. My first thought was one of joy,—"Peace at any price! Let sleeping dogs lie!" But I soon saw that Tom was watching me, to see if he could, as he would say, "put one over." All the other boys were on the watch too, furtively.

I let some time pass. Tom became so absorbed in his book he forgot to look at me. I began a slow tour around the room, drawing up finally in martial array in the rear of the enemy. The silence in that room cried aloud. Tom felt it and looked up.

"Give me the book, Tom!"

I glanced at it. It was a Sherlock Holmes story. Involuntarily, I exclaimed:

"That's a *good* story, don't you think so?"

"Yes, *ma'am.*"

"I'll put in on my desk. You can get it after school. Believe me, it will be all I can do to keep away from it, Tom. But I suppose the Board of Education is not paying me to read stories in school? Eh, Tom?"

Tom took the hint, got out his textbook and worked. The next morning he brought the book back to me, and mumbled shyly: "I read 'em all las' night. Thought p'r'aps you'd like to read 'em, too!"

I read that book with all my soul. This was the situation as I saw it. I had a gang on my hands; I was a new teacher. I must win them, or they would break me. How could I get hold of them? Evidently and best—through their interests, and the Sherlock Holmes detective story was one of these interests.

Curiously enough, these stories were founded on deductive reasoning and analysis,—and so is geometry. Why were they attracted to the one and repelled by the other? Were there any "identical elements," as the psychologist would say, that

could be extracted and held up to the light? One might answer that the Holmes stories present a mystery, but it is equally evident that the same is true of some geometry problems,—as all parents will attest. And so the questions came to me: Can I take a Sherlock Holmes story and work it out mathematically? Can I take a geometry problem and work it out in Holmes' best style? Can I show that the reasoning, deductive in each case, is intrinsically the same? Can I thus use the detective instinct in a worth-while cause?

I turned to Mr. Holmes for pointers.

"Sherlock Holmes closed his eyes and placed his elbows on the arms of his chair, his finger tips together.

"'The ideal reasoner,' he remarked, 'would, when he is once shown a single fact in all its bearings, deduce from it not only all the chain of events which led up to it, but also the results which would follow it. The observer who has thoroughly understood one link in a series of incidents, should be able to state accurately all the other ones,—both the before and the after. We have not yet begun to grasp the results to which pure reason alone can attain. Problems may be solved in the study which have baffled all those who have sought a solution by the aid of their senses. To carry the art, however, to its highest pitch, it is necessary that the reasoner should be able to utilize all the facts which have come to his knowledge. This in itself implies, as you will readily see, a possession of all knowledge, which even in these days of free education and encyclopedias, is a somewhat rare accomplishment. It is not so impossible, however, that a man should possess all knowledge which is likely to be useful to him in his particular work, and this I have endeavored in my own case to do. If I remember rightly, Watson, you on one occasion, in the early days of our friendship, defined my limits in a very precise fashion.'

"Holmes smiled. 'Well! I say again that a man should keep his little brain attic stocked with all the furniture he is likely to use, and the rest he can put away in the lumber-room of his library where he can get it if he wants it. It is of the highest importance in the art of detection to be able to recognize, out of a number of facts, which are the incidental, and which the vital. Furthermore, it is an old maxim of mine,

that when you have excluded the *impossible*, whatever remains, *however improbable*, must be the truth.'"

In brief, these are the three points Sherlock Holmes makes in his statement:

1. Marshal all related facts.

2. Select the essentials, by rejecting the incidental and by excluding the impossible.

3. Study the improbable.

With these guides in mind we are now in the position to study the "Boscombe Valley Mystery."* Briefly the facts are these: James McCarthy was found dying near the edge of Boscombe pool, a small pond nearly surrounded by trees, and about 300 yards from the road. His son gave the alarm, stating that he had heard his father cry out and had gone to his assistance, reaching him a moment before he died. On the ground near the pool were a gun, a large stone, and several footprints. The blow from which McCarthy died was on the back of the head.

Clearly now the gun, the stone, the footprints and the blow constitute the data or given. The murder is the result or conclusion. It becomes the detective's duty to connect cause and effect by an unassailable mathematically correct line of reasoning.

It is evident that there are two plans of attack—either on the cause, or on the effect. Let us consider first the data, and see what they have to tell us, and whither they lead us.

There are two possibilities—murder or suicide. Let us assume the man was murdered. In that case the gun might be the murderer's, or the murdered man's. The stone mentioned was lying on the grass—evidently brought to the pool—but by which one? The question looks hopeless so far; which way can we turn for more light?

The blow from which the man died was on the back of the head under the left ear. This points to the conjecture or hypothesis that it was given by someone from the rear, and that the implement of death was thrown by a strong hand, very likely the left one. But is it reasonable to expect the detective, in

* From "The Adventures of Sherlock Holmes," by Sir Arthur Conan Doyle.

his effort to run down the criminal, to examine all the left-handed men in the world and get an alibi from each?

What about the footprints? Do they lead anywhere? Examination reveals the fact that the left print is deeper than the right one. This argues that the murderer limped in his right leg. Must the detective discuss the question with all men who limp in their right legs?

What is the situation now? Let us put it into the form of a little chart.

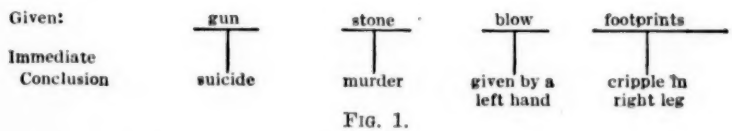


FIG. 1.

The data have all been studied, much has been learned from them, but the detective has arrived at an impasse of insuperable difficulties. The survey is too broad. It is world-wide. It looks hopeless. And so indeed feel the boy and girl when confronted with a geometry problem. Though they know well what they have to work with, and know well what they want, they have not always sufficient imagination nor experience to penetrate the darkness that lies between the two, and thus make the necessary links in their reasoning. In their hopeless groping and waiting for the inspiration that does not come they lose courage and interest and "give up."

Let us, therefore, make our attack from the other end. Let us reason from effect back to cause. We can in this way greatly *restrict* our problem. We can, as said, reason from one effect to its possible immediate causes. According to Holmes, we shall marshal our facts, reject the impossible and incidental, and select an essential cause. Having found a plausible one, we shall consider it in its turn an effect, and search for the immediate causes leading to it. By repeating this process several times we shall eventually arrive at the first cause. Thus by the various stages of analysis we shall unravel the mystery. Having done this by analysis, we can reverse the process and build up the entire story or proof by synthesis.

The murdered man, therefore, became Holmes' first con-

sideration, and his natural question was: "Who were the last people to see McCarthy alive?" Inquiry revealed these last witnesses to be an old man, a game keeper, a young girl, and a servant. Since their testimony *antedates* the murder, we can make a little *chronological* scheme, writing them first thus:

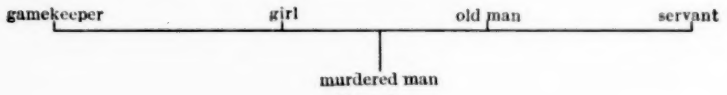


FIG. 2.

All the facts are now marshalled. The old man can be eliminated, as he could only say he saw McCarthy on the road. The gamekeeper and the girl said they saw McCarthy going toward the pool, but that they saw the son follow soon after. The servant said his master mentioned an appointment with someone but did not say where nor with whom.

Our scheme now looks like this:

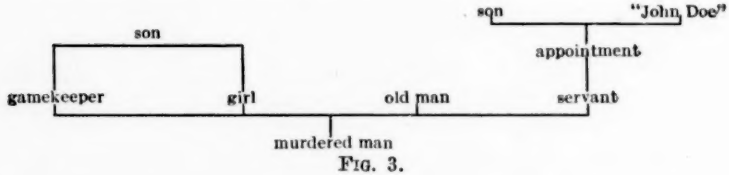


FIG. 3.

When questioned the son said that on that afternoon he intended calling at a house some distance down the road that led past the pool. When nearly opposite to it, he heard the call "cooee," which was the signal he and his father used to call each other. He went over, talked with his father, who seemed surprised to see him and to know that he had returned from the city. He had some words with his father and then went on his way. When he had passed the bend in the road and was out of sight of the pool he heard a cry. Thinking that harm had come to his father, he ran back, arriving just in time to hold the dying man in his arms and hear him say: "a rat." He did not know the significance of the word rat. The situation now is this: The son confesses a connection with the word "cooee," but denies any understanding of the dying man's last words. It is but natural that such words under such tragic

circumstances would be of something relative to the murderer or to the manner of death.

At this point, therefore, it is only fair to keep an open mind concerning the son's probable guilt and to discuss the possibility of a stranger's having done the deed. Gathering up the threads our scheme now looks like this:

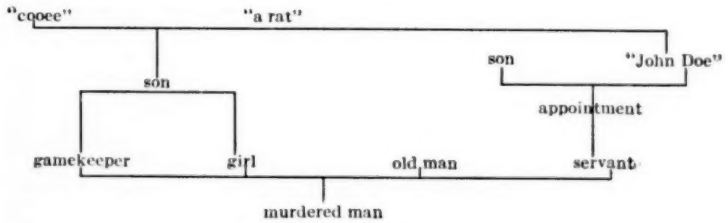


FIG. 4.

Holmes then stated that "cooee" is a well-known call among Australian miners; also that in Australia there is a town called Ballarat, and that it would have been a most natural thing for the dying man to have named his murderer as "so-and-so of Ballarat." "Cooee" and "Ballarat" thus point to an acquaintance whom McCarthy had known in his early days in Australia. Following this line of thought, there are two alternatives: Is the Australian acquaintance a neighbor or a visitor? Inquiry revealed the fact that there was a near neighbor by the name of Turner who had known McCarthy in the early days in Australia. So our chronological scheme is now like this:

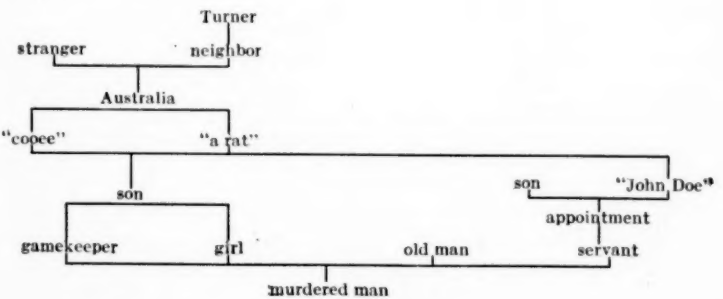


FIG. 5.

The next question is: "Does Turner check up in any way with the data?" The gun was proved to be McCarthy's from the evidence of the servant. The big stone might have been there several days, although the grass under it was still green. *But* Turner was a powerfully built man, used his left hand and limped in his right leg. All of our evidence can now be represented diagrammatically as follows, the complete synthetic proof developing along the route of the dotted line.

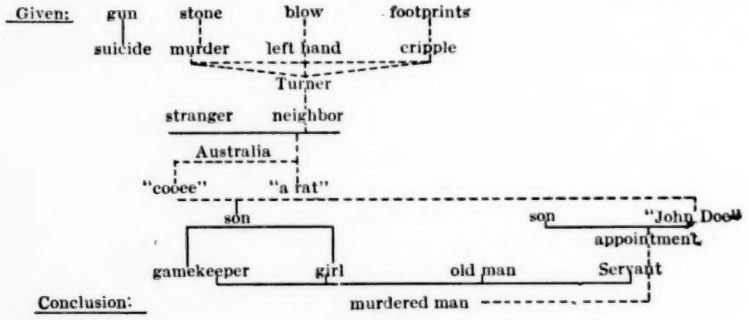


FIG. 6.

By analysis and by reasoning from effect to cause, Holmes arrived at the first cause. Reversing the process, and following the dotted line it is possible to build up the entire story synthetically. This exact process can be carried out in a geometry problem, and thus be made a "matter of transfer." In what follows notice that the three rules of Holmes are adhered to. *At every step* all the facts are *marshalled*, and *considered* as possible immediate causes. The *essentials* are selected through discussion, and by the rejection of some points that are incidental or impossible when considered in the light of the given. Lastly, the "*improbable*" proves to be probable because it checks up with the data.

In putting these schemes on the board, in the introduction to demonstrative geometry, it is best to write from the bottom up, building up the structure as each new step is found. When finished, if written in this way, the synthetic proof is in the proper order, and the pupil does not have to make a new orientation to state it in the usual form. This saves him from getting "mixed up," as he puts it.

Let us now proceed to solve a geometry problem by Holmes' own methods.

Given: An isosceles triangle $ABC$, in which $CD$ is the altitude to $AB$, $AE$ is the altitude to $BC$ and $AE = CD$.

To prove: $\angle ACD = \angle CAE$.

Write the given at the top of the board or paper, and the conclusion at the bottom—leaving the wide space between to be filled in. Since the immediate focus of attention is now on the conclusion, *i.e.*, proving the equality of two angles, the obvious and only question is: What situations produce equal angles? Experience and memory evolve a scheme in part like this:

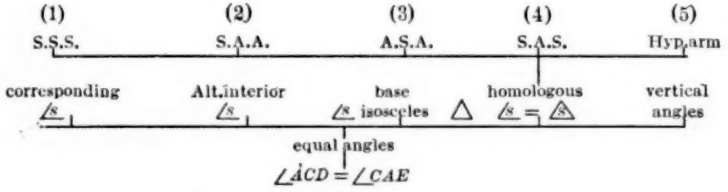


FIG. 7.

Discussion rejects (1) and (2) for the obvious reason that there are no parallel lines. A study of (3) and the drawing reveals the fact that the required angles are not the base angles of the given isosceles triangle, and though triangle $AOC$ looks isosceles, there is no way of proving it so from our data. Also (5) is out of the question. Thus (4) appears to be the only way out—improbable though it may appear to the novice. The next question is, therefore: Under what conditions are triangles equal? The growing scheme on the board appears thus:

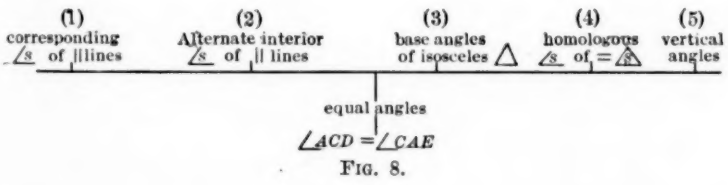


FIG. 8.

The comparison of the hypothesis and the triangles $ACD$ and $CAE$ with (1) discloses the fact that nothing is known

concerning the third sides; while the examination of the drawing shows that the three elements required for the equality of the triangle are not arranged in the proper relation to admit (3) and (4). These elements, however, are arranged in the order of (2) or (5), hence it is readily seen that there are two solutions, and our scheme at this point, checking up with hypothesis, is as follows: We have come from conclusion to data, and can now reverse our steps, reading the synthetic proof as it develops along the dotted line.

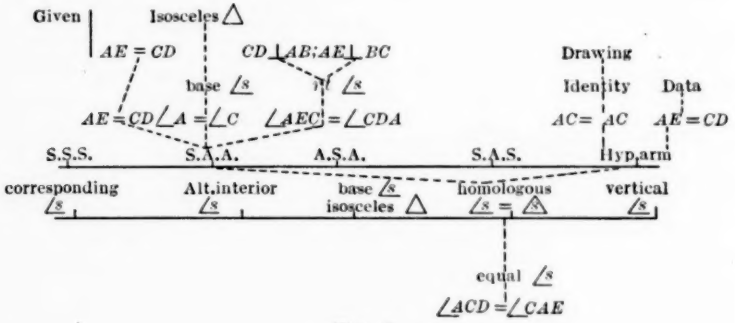


FIG. 9.

Space need not be taken here to write out the synthetic proof, as it is so easily seen by reading the diagram in the usual direction from the top down. This method of attack to a high degree removes vagueness and uncertainty, substituting a definite rule which invites progress. It furnishes a constant drill in the *organization* of thought as well as of subject matter and thus makes for clear thinking.

So much for the methods of the great detective. As for the man himself—the student's prototype—Dr. Watson, says: "Sherlock Holmes was a man, who, when he had an unsolved problem on his mind, would go for days without rest, turning it over in his mind, re-arranging his facts, looking at it from every point of view until he had fathomed it, or convinced himself that his data were insufficient."

BLANCHE B. HEDGES.

DEARBORN-MORGAN SCHOOL,
ORANGE, N. J.

# REMARKS ON THE REPORT OF THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS ON COLLEGE ENTRANCE REQUIREMENTS.

I. By E. R. Hedrick, University of Missouri.

The report of the National Committee on College Entrance Requirements will seem at first to violate some of the traditions that we have held dear. It is my purpose to discuss briefly these changes and to try to evaluate the effects on collegiate mathematics.

Formerly, the teachers of mathematics in colleges thought of entrance examinations wholly from the standpoint of preparation for their own courses, and it was assumed that the work in mathematics in secondary schools would be shaped to prepare students for those courses. It is probably fair to say that this assumption is no longer the basis for college entrance examinations. It is generally recognized that the work of a high school should be determined by the best interests of those who constitute the majority in those schools, and it seems highly probable that college mathematicians can benefit mathematical instruction in the country far more by taking this attitude.

Wherever this view is accepted, an effort will certainly be made to revise the college courses in mathematics so that they will be adapted to the needs of students entering from secondary schools after those students have had courses based on the suggestions of the National Committee itself, or on other good authority.

While this should be the case in the courses in the Department of Mathematics, it is clear that courses in the other scientific departments can not be changed so readily. I understand that it is for this reason that the committee attempted to find out what topics in elementary mathematics were used most frequently by these other scientific departments. It seems to me that collegiate courses in mathematics could be changed in

the manner indicated above, and I believe that the change will be beneficial and stimulating, rather than otherwise. It is true the collegiate courses have become formalized no less than secondary courses, and that they stand in need of the reorganization which will be forced if this program is carried out.

It appears that the committee formulated this report without consciously considering its own previous report on Mathematical Requirements in Secondary Schools. This should have been done, perhaps, because it would be fair to assume that candidates for college entrance could make special preparation in the last years of the high school, if the requirements of college entrance differed materially from courses required in the high school for all students.

Perhaps the most surprising feature of the entire report is that it does agree in practically every essential point with the recommendations made in the report on Mathematical Requirements in Secondary Schools. I have just pointed out that this was not done by intention. Is it possible that this surprising result should have come about by accident and that that accident should have happened uniformly in the same sense with respect to a number of different topics? I believe that I can point out that the result is not at all accidental.

In the previous report for Secondary Schools, the committee announced as one of its fundamental principles that the topics selected should be based, not upon the requirements of courses that are to follow in later years, but rather upon the development of a better appreciation of civilization and a better understanding of life about us. The Report on College Entrance is not based on this principle, but on the needs of modern science in all the great departments of human knowledge. Is it simply by accident, then, that a topic discarded because it showed no immediate gain in the mastery of life, turns out to be of little or no consequence for later courses in the fundamental sciences? Or that a topic, included because it would add to the student's grasp of civilization, should turn out to be of considerable consequence in higher courses in science? It seems to me that the surprising agreement between these two Reports in spite of their different points of departure, is by no means accidental. If we honestly adhere to the basis of choice emphasized by the committee in its Secondary School

Report, we shall certainly secure topics that have direct connections with life and with world-activities. It is certainly to be presumed that higher courses in fundamental sciences will themselves deal with topics that affect real life and the activities of the outer world. If they do, it is not to be wondered at that students, trained toward greater control of life-activities, will also find themselves best equipped to carry on these higher courses. It has proved true in the past that a choice of topics based upon alleged needs of future courses has led, frequently, to a choice of meaningless manipulation, and of processes of no value, either at the time or at any future time. Hence I should prefer to take my chance with topics of known immediate value toward the appreciation of life and of civilization, and I would hope that these would prove to be most useful in collegiate courses, if those collegiate courses are themselves in tune with life.

II. By H. D. Gaylord, Brown and Nichols School, Cambridge, Mass.

The reports of the National Committee on Mathematical Requirements have given evidence of so much careful investigation and analysis that no teacher of the subject should fail to make a study of each forthcoming report. The committee gives us a preliminary report on College Entrance Requirements, which, although it is intended primarily for the guidance of those whose students expect to enter college, contains so much of value for all who teach either algebra or geometry that it is well worth careful consideration by all high-school teachers of mathematics.

This report makes a most helpful analysis of the relative values of the various topics in algebra and clears up much of the vagueness which the teacher is apt to feel in regard to the importance of certain parts of the course. It also points out the fact that the relative values set on these topics by the teachers of the social sciences and the teachers of the physical sciences in colleges are in striking accord with the other reports of the committee on programs in mathematics for junior and senior high-school students who do not intend to enter college.

This testimony from so many other fields that certain topics

in secondary school mathematics are essential to the proper appreciation of the material in those fields should give teachers of mathematics a better understanding of the remarkable part which mathematics plays in the whole structure of learning. If the teachers of mathematics can sufficiently detach themselves from the routine drill which is necessary for efficient preparation for the tests set by formal examination boards to view the subject of mathematics in a broad way, they will find in the first three topics of the minor requirement in elementary algebra much material which can be made useful in bringing the student into touch with the human side of mathematics.

Many of us have looked upon the formulas borrowed from the sciences and the engineering textbooks as either too difficult for the pupils to understand or as containing very little algebraic meat. Hence we have plunged ahead into the formal parts of the subject in order to enable the student to acquire quickly the ability to perform the six or eight stunts which are usually found on examination. The simple formula, the table of changing values, and the graph derived from such formulas or tables give an opportunity to link mathematics with the real life of the world and at the same time teach some of the great underlying mathematical facts. These topics which offer plentiful opportunity for practice in numerical computation should furnish an easy means of transition from arithmetic to algebra and a sufficient review of the fundamentals of arithmetic. They should offer also good illustrations of the need for the more generalized methods of algebra to replace the individual and particular applications of arithmetical numbers.

Although the report makes no attempt to prescribe a teaching order of topics, the suggestiveness of placing formulas, variables, and graphs at the beginning of the course before any attempt is made to introduce such formal algebraic conceptions as negative numbers and the methods of manipulation which are needed to make the four fundamental operations perfectly general should induce serious thought on the part of teachers. It suggests that, although students of high-school age are not philosophers and are not over-anxious to find reasonableness and logic in the development of the course, yet the need for a more general form of symbolism such as that used in algebra

should be shown to them through a series of problems which are closely enough connected with the real affairs of life to avoid giving them the feeling that algebra is an artificial system devised mainly for amusement or torture. The fact that algebra is generalized arithmetic makes its most compelling appeal to the student when he realizes that there is a real need for such a generalized arithmetic.

That part of the report which deals with the content of entrance examinations is especially valuable to the teacher of algebra because it offers so many suggestions in regard to the underlying objects of the course. It is very desirable that any course should have, in the mind of the teacher, a clearly defined purpose as a part of the whole educational plan of the school. The recommendations of this committee and of the committee appointed by the College Entrance Examination Board bring clearly before us the question of the results which we should expect to accomplish in the minds of the students who take the course in algebra. The entrance examination should be a test of the accomplishment of this result, and it is therefore highly desirable that the teacher should have constantly before him the real purpose of the course and continually test himself and his class in regard to their progress toward this end.

These recommendations suggest that "fully one third" of the questions should be of such a fundamental nature that all faithful students can be expected to answer them satisfactorily. If in connection with this recommendation the National Committee had set forth in a paragraph or two what constitutes that fundamental part of the course or had given a definite statement of the fundamental purpose of the course in algebra with a brief mention of the topics which bring out this purpose, this part of the report would have been even more helpful to teachers.

This broad purpose is suggested when the committee recommends that "The examination as a whole should, as far as practicable, reflect the principle that algebraic technique is a means to an end, and not an end in itself." We are left somewhat in the dark, however, as to what that end is. It is true that such a paragraph occurs on pages 2 and 3 of the report entitled The Reorganization of the First Courses in Secondary

Mathematics. This statement of the fundamental aims in the teaching of algebra cannot be too often repeated, and it would add much to the usefulness of this report if a place could be found for such a declaration of aims in connection with this warning to avoid excessive practice in algebraic technique and manipulation.

The report on First Courses says, "On the side of Algebra, the ability to understand its language and to use it intelligently, the ability to analyze a problem, to formulate it mathematically, and to interpret the result must be dominant aims. Drill in manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and of probable applications either in common life or in subsequent mathematics." This statement of aims should be so clearly a part of every teacher's working plan that no undue amount of time would be spent on any topic in the hope of extending the student's knowledge to a point beyond which it will be useful in accomplishing these "dominant aims." Then the entrance examination may well test the accomplishments of the student on this basis by means of "simple and difficult questions testing the candidate's ability to apply the principles of the subject."

## THE PROBLEM OF HOME WORK PAPERS.

It is the practice of many teachers of high-school mathematics, and particularly of beginning algebra, to require their students to hand in daily home work papers. No useful purpose is served by this unless the papers are carefully corrected and returned to the pupils at the next recitation. The careful daily correction of such a large number of papers, however, requires time and such a large amount of energy that the teacher who attempts it has no time or energy left for more important things. The teacher of high-school mathematics today must devote considerable time to professional improvement, to the reading of magazines such as THE MATHEMATICS TEACHER and of educational journals of a more general character, and to the study of the larger problems of his profession. Further, he must have time to carefully plan his daily work and to study and solve his individual problems. Work of this kind demands time and energy and is much more important than the grind and drudgery of correcting home work papers.

Home work papers are usually intended to serve two purposes: (*a*) to enable the teacher to keep constantly in touch with the work done by individual pupils and thus to detect errors and misunderstandings almost as soon as they occur, and (*b*) to give the teacher evidence that the practice work assigned to be done by the pupils outside of the class hour has been performed.

It is very important, particularly when starting a new type of work, to keep in close touch with the work being done by individual pupils so as to detect and correct mistakes at once. Home work papers are one means of doing this; they are not the only means, however, nor are they the best. They show only the finished product and do not always make clear the mental processes by which that finished product was obtained. In other words they show the students' mistakes but too often do not show the cause of those mistakes. Further, the papers do not always represent the work of the student, who often

receives help at home (and home help too often means that the parents or an older brother or sister have really done the work), or perhaps copies the work directly from some other student. Home work papers, by themselves, are a very wasteful and unsatisfactory means of locating the students' difficulties. The only means of really doing this effectively is class room work, at their seats or at the blackboard, under the direct supervision of the teacher. The class period should be a work period and not a recitation. A wide-awake, skilful teacher can discover more about an individual student's difficulties and ability in one supervised work period than by correcting home work papers every day for a week. The ideal way to teach mathematics is to have a longer class period and to have all of the work done in class under the supervision of the teacher. Even where this is not possible, however, most of the class period should be devoted to supervised work on new material.

If the class periods are so short that all of the work can not be done in class, and assignments for outside preparation must be made, it is necessary to check up on the work in some way to be sure that all of the students are preparing the assignments. Having the students hand in home work papers is, again, one means of doing this. It is not a very effective means, however, as the student who has not prepared the assignment almost invariably copies the work from some other member of the class. After all, the best evidence that the teacher can have that the student has done the work assigned for home preparation is the ability of that student to do more work of the same kind in class. In fact, it is much more important to know what a student can do, unassisted in class, than to know what he has done outside of class, perhaps with more or less assistance.

In addition to the two purposes already mentioned, home-work papers are sometimes used in another way. Some teachers grade the daily papers and use the marks given in determining the final standing of the students. This is bad practice in several ways. In the first place it encourages cheating. If the students know that their daily home work papers are used in determining their final standing the temptation to hand in a good paper, even if they must copy it from some other student,

is very strong. In the second place it puts the slow pupil at an unfair disadvantage. The student who learns slowly, who does not grasp a point immediately but who by patient work finally masters the subject, is penalized unfairly if the mistakes he makes in the early stages of each new type of work are counted against him. Students should learn that it is no disgrace to make mistakes, that the only disgrace is to keep on making the same mistake time after time. They should see that it is often by making mistakes and correcting them that they grow and laern. For this reason the best basis for awarding a mark to a student is the teacher's estimate of the work finally done by that student, based on close observation of the work done by the student in class and upon the results of short tests given at frequent intervals and covering small units of the work.

If the point of view taken in the preceding discussion is correct, the skilful teacher can, without the drudgery of correcting daily home work papers, accomplish more by some such method as the following than could possibly be accomplished by such papers.

I. A few minutes should be taken at the beginning of the class period to check up on the outside work assigned. This can usually be done rapidly by reading and comparing results. As soon as the individual students find out which problems they have failed to solve correctly, they should be sent to the board to try these problems again under the watchful eye of the teacher so that he can locate and help them overcome their difficulties. Students who have no difficulty with any of the assigned work, and the others as soon as their difficulties are mastered, should be given new material to work with. It is true that pupils may not always report difficulties, but if they do not they will get into similar trouble with the new work attempted in class, which will serve the teacher's purpose equally as well.

II. If the teacher feels the need he can have papers handed in occasionally, particularly when taking up a new type of work. If the pupils know that their home work papers may be called for at any time without warning it will be just as effective in forcing them to prepare the daily assignments as

if the papers are handed in each day. Whenever the papers are handed in they should be corrected and returned at the next class period, at which time the pupils should be required to work over, under the supervision of the teacher, all problems on which they failed.

J. R. Overman.

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## TEACHING INCOMMENSURABLES

The teaching of incommensurables is largely a matter of expediency. If a class recognizes that similar quantities may or may not have a common measure, and if it questions whether theorems true in the first case are also true in the second, then that class is in a position to follow the proof of the incommensurable cases with appreciation, even though no one of the class can reproduce it independently. The fact that the proof exists and that the pupils have followed it is the important thing.

If, on the other hand, the class is too immature to realize that incommensurables do exist, the proof of such cases is best postponed until the geometry is reviewed. A theorem that seems useless on the first reading of the subject, often slips into place on the second.

Unless Legendre's proof be used, the notion of incommensurables implies that of limits. To my mind this concept can well be introduced here. It develops the idea of variables implied in the work on formulæ and the graphic treatment of equations; it leads to an extended treatment in the Calculus. The idea of a variable that tends toward a limit is familiar—the ripple when a stone is dropped into a pond, the creation of a partial vacuum, the disintegration of radium, the height of a growing boy. All that really is needed is to clarify our notions of variables by defining the conditions under which they may be said to approach limits.

Geometric illustrations of this definition have the great advantage of resting on accepted postulates; for many of the so-called practical illustrations require more time than the illustration justifies for the preliminary discussion that sets up the problem.

In a search for a quantitative geometric illustration of limits I came upon the following. The figure is used in another connection by Maxwell in his Non-Euclidean Geometry p. 27.

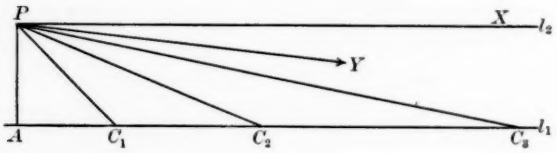


FIG. 1.

Suppose $l_1$ and $l_2$ are each perpendicular to $PA$.

$$\text{Let } AC_1 = PA,$$
$$C_1C_2 = PC_1,$$
$$C_2C_3 = PC_2, \text{ etc.}$$

There is no limit to the number of points $C_1$, $C_2$, $C_3$, for a line may be extended for any required length—in particular to the length $PC_4$.

This gives an infinite series of points on $l_1$ and also an infinite series of lines $PC_4$ each greater than the preceding one. The corresponding segments $AC_1$, $AC_2$ therefore increase without limit.

Here, then, is one of our illustrations.

Corresponding to each point $C_4$ there is an angle $PC_4A$. An infinite series of these points gives rise to an infinite series of these angles and each angle is one half of its predecessor

Each of these angles is complementary to the angle $APC_4$.

If the angle $PC_4A$ becomes equal to zero, the angle $APC_4 = 90°$ and $PC_4$ is parallel to $AC_4$, which is impossible. Therefore the angle $PC_4A$ cannot become equal to zero, and by the principle of continuity it cannot become less than zero. It is evident that however small an angle $YPX$ we may draw, some line $PC_4$ of the series can be found longer than $PY$. Then the angle $PC_4A$ will be less than the angle $PYA$. Thus the angle $PCA$ approaches zero as its limit, and similarly the angle $APC$ approaches 90° as its limit.

The angle $PCA$ equals the angle $CPX$.

Now if the limits of these two angles are not equal, one of two things must be true. Either $l_1$ has ceased to be parallel to $l_2$ which is contrary to our hypothesis; or alternate interior angles of parallel lines have ceased to be equal, which is inconceivable. This figure, then, illustrates a variable which be-

comes infinite, a variable that approaches zero, and two constantly equal variables approaching their limits.

This detail cannot, of course, be given to a class in toto, but the possibilities of the figure appeal to a class and much of the work can be developed by the pupils in a short period. It also helps in the realization of the essential normalness of our definition of a limit.

Legendre's proof of incommensurable cases may be interesting in this connection. It is an exquisite example of a reductio ad absurdum. He applies this proof to the rectangles having equal altitudes and to the central angle proposition. He proves the proportional division of the sides of a triangle by a different method, although the method of the other cases may readily be extended to this one.

Let us take the proposition of the rectangles having equal altitudes (Legendre Book III., prop. 3).

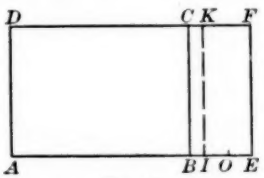


FIG. 2.

If the bases $AB$ and $AE$ of the rectangles $ABCD$, $AEFD$ were commensurable we would know

$$\frac{ABCD}{AEFD} = \frac{AB}{AE}$$

If $AB$ and $AE$ have no common measure, it may be necessary to substitute for $AB$ a quantity $AO$ to have a true proportion.

Then (1) $\dfrac{ABCD}{AEFD} = \dfrac{AO}{AE}$

$AO$ may be greater or less than $AB$.

Suppose $AO$ is greater than $AB$. Then $O$ lies on $AE$ between $B$ and $E$.

Take some unit less than $BO$ which will be contained a whole number of times in $AE$. Mark this unit along $AE$. At least

one point of division, $I$, will come between $B$ and O. At $I$ draw $IK \perp AE$.

$AI$ and $AE$ are commensurable by construction.

$\therefore$ (2) $\quad \frac{AIKD}{AEFD} = \frac{AI}{AE}$.

Divide (1) by (2) $\frac{ABCD}{AIKD} = \frac{AO}{AI}$,

*but* the first of these ratios is less than one, the second greater than one, therefore the proportion is untrue and the hypothesis that $AO$ is greater than $AB$ is absurd. Similarly $AO$ cannot be less than $AB$.

$\therefore AO = AB$ and, substituting in (1),

$$\frac{ABCD}{AEFD} = \frac{AB}{AE}.$$

VERA SANFORD

THE LINCOLN SCHOOL,
NEW YORK CITY.

## ROUND TABLE DISCUSSION

This department of the TEACHER is conducted by Eugene R. Smith, The Park School, Baltimore, Md. All correspondence should be addressed to Mr. Smith. Readers are invited to submit questions or answers in the field of their special interests, or to discuss briefly any topics of interest, whether suggested by articles in the TEACHER or by personal experience. The editor in charge will refer questions to persons specially qualified to answer them.

Q. 1. I note that the report of the National Committee attaches considerable importance to certain elementary topics in statistics. Where can a teacher who has not had the advantage of a course in this subject obtain information regarding the fundamental principles of the science of statistics? J. H. A.

Of the books on statistics the writer prefers to recommend "Statistical Methods Applied to Education" by H. O. Rugg, Houghton Mifflin Company, 1917. This book presupposes very little mathematical training. The principles are discussed and illustrated at much greater length than in the average book. For a most concise statement without any special field of application, one can read with profit, "The Elements of Statistical Method," by W. I. King, The Macmillan Company. A slightly larger book in which not so many formulæ are assumed and in which the calculus is not employed, is "Elements of Statistics," by A. L. Bowley, Charles Scribner's Sons.

Bowley is to be preferred to King for those with mathematical training.

[*Note:* Can some one suggest other books dealing with statistical methods that can be taught to children?]

Q. 3. I note that the National Committee has recommended that a rather large amount of intuitive geometry, considerable algebra and some numerical trigonometry, be given before demonstrative geometry. Assuming that this is done, will there be any justification for correlation in the demonstrative geometry? H. B. K.

When a large amount of intuitive geometry and considerable algebra precedes demonstrative geometry, it may be assumed that the pupil knows much about the informational material usually offered in plane geometry and that he has had considerable training in drawing correct inferences from given or assumed conditions. A mere repetition of these experiences would be a waste of time, and it should, therefore, not be necessary to spend a full year on the study of plane geometry. For a year's work we may add algebra, or solid geometry to the work which is purely demonstrative geometry. Such a course would give ample opportunity for correlation.

However, if a teacher prefers to give a brief course in intensive study of plane geometry, where training in reasoning is one of the principal aims, correlation will be justified wherever it helps the pupil to gain better understanding and mastery. For example, the use of algebraic symbols greatly simplifies some of the proofs. The use of the equation in theorems expressing functional relations proves exceedingly helpful. This applies especially to theorems on angles and arcs; sides of right, acute, and obtuse angled triangles; segments of chords and secants; medians and sides; radii of inscribed and circumscribed circles and the sides of regular polygons; proportions in similar figures; segments of transversals cut by parallel lines.

The applications of these theorems lead to equations of the first and second degree, involve the fundamental algebraic operations and therefore give wide opportunity for correlating geometry with algebra.

Furthermore, the pupil's knowledge of trigonometry enables him to simplify a number of proofs. For example, the relations between the side of a regular polygon and the radii of the inscribed and circumscribed circles are easily established by use of the trigonometric functions.

In all formulas every possible opportunity should be used for arithmetical computation.

All these experiences must lead the pupil to a better understanding than can be attained by specializing on logical geometry to the exclusion of all related material. The question of the writer should, therefore, be answered in the affirmative.

E. R. Breslich.

The School of Education,
University of Chicago.

Q. 5. What are the important qualifications to be considered in choosing the head of a department of mathematics?

"If you were in a position to select the head of a mathematics department from a group of mathematics teachers, what qualifications would you look for? Will you write them down in order of importance, the most important first?"

These questions were asked of our nine high school teachers of mathematics. Below you will find their answers. The numbers after a certain qualification are the ranks the teachers gave to it.

Knowledge of the subject—based on scholarship, degrees, years of experience, etc. 1, 2, 4, 2, 1, 1, 1,

Success as a teacher—1, 3, 4, 3, 1.

Executive ability—2, 4, 3, 1.

—Be able to recognize good teaching in others and to be able to commend without flattering. Be able to recognize poor teaching in others and to be able to criticise without offending. Efficiency as a teacher sufficient to inspire fellow teachers and to create a high standard of work required of students.

Decisive force—1, 5, 3.

—Manifested toward pupils and those in authority. Sufficient stamina to support teachers in any justifiable cause. He should have sufficient aggressiveness to stand out for the rights of the department.

Standing in the community—2—moral, social, good citizenship.

Leadership—1.

—He must be awake to the spirit of the times and keep himself informed as to what other schools are doing.

Ability to solve pedagogical problems—4, 3.

Character and personality—1, 2, 1.

Tact—5, 2. True dignity—2.

Sympathy—6. Interest—3. Magnanimity—2.

From the answers we notice that the majority place scholarship first—scholarship receiving four firsts. Successful teaching received two firsts and several of the others one each. The one who gave scholarship 4th place gave executive ability first, and good teaching third. The one who gave character and personality first gave scholarship second and the one who gave leadership first gave scholarship second.

No definite conclusions can be drawn from such a few answers but they are valuable as suggestive of what qualifications each teacher might look toward.

FRED H. CROSINGER.

HEAD OF MATHEMATICS DEPARTMENT,
FORT WAYNE, INDIANA.

Having met this problem in practice I can define rather accurately my own way of choosing.

In the first place no one could be considered unless he or she was of good character and had adequate training and successful experience along mathematical lines. That fact is so evident that it does not enter into the real problem, which is to choose the best one from those who have these fundamental qualifications.

Th most important attribute for which to look seems to me to be the ability to win confidence and cooperation. This is all important, for without it one cannot get real team work from his teachers or inspire his pupils.

I should also demand open-mindedness. The man or woman who cannot see the good in others ways of doing good things, who is not always alert for better material, texts and methods, will never make a real success. If in addition, there is enough initiative and originality to start new things for himself, he belongs to the elect.

One good criterion by which to judge one aspiring to promotion is this: Is he such a success where he is that he has outgrown the job? The one who has "burst the seams" of the suit he is wearing may fairly demand a larger one.

One could not meet the requirements I have outlined without having most of the qualities mentioned in the preceding answer, but the mind of the one choosing should, I think, be centered on a few inclusive characteristics rather than on too many details.

E. R. S.

(How do other teachers or executives feel about this?—EDITOR.)

PROBLEMS FOR DISCUSSION.

Q. 6. I should like to know whether any Junior High Schools have introduced the Slide rule? B. B.

Q. 7. I should like to know whether the readers of the TEACHER find historical notes of historical material in general, of any real value for their pupils? R. P.

Q. 8. How should supervised study be administered in the teaching of junior high-school mathematics? P. D. E.

Q. 9. Our Board of Education is interested in finding out the essential qualifications for teachers of Junior High School Mathematics. Has any report been made on this? If not, what should be recommended?

The Editor urges readers of the TEACHER to join in the discussion of the points raised and to contribute questions for discussion.

## NEWS AND NOTES.

The mathematics teachers of Kansas City are cooperating with the Board of Education in determining the qualifications which should be required of mathematics teachers. Mr. William A. Luby, of the Kansas City Junior College, will discuss this action in an early issue of the TEACHER.

Through the activities of the Mathematics Club of St. Louis and vicinity there has been a remarkable increase of interest and professional spirit in the problems of mathematical education. Alfred Davis, who is president of the Club, and Vincent W. Brown, manager International Accountants Society, have frequently used the daily press of that city as a means of bringing to the public a better understanding and appreciation of the value of mathematics in modern education.

We illustrate the type of discussion which has appeared from time to time in the daily papers by quoting from one of Mr. Mr. Davis's articles which appeared in the *St. Louis Daily Globe Democrat*, Sunday, January 9, 1921:

Most of the courses available to high school students of St. Louis are elective. That is, the student is responsible for selecting the subjects he is to study. Of course, the teacher and the parent may advise, but unless certain courses are chosen, or certain colleges to which admission is desired require them, some important studies may be entirely omitted from the program. Some pupils will dodge studies which they need because they wish an easy time and their aim is to get a diploma at the least cost. Others will miss important studies because they lack the knowledge and the experience necessary to make a wise choice. It is quite a common experience to have pupils return to take high school studies which they have omitted because they have found the way to success blocked from this lack. This is especially true of mathematics. Pupils may and do graduate from the high schools of the city without taking mathematics and later realize the great mistake they made.

It is highly desirable that the pupil should have all possible

information about a subject if he is to decide whether he shall study it or not. Unfortunately one cannot learn much about mathematics by hearing someone talk about it. To appreciate it and to know its value one must study it under expert teaching. It is for this reason that seemingly arbitrary statements must be made about the subject, and we must bring to the aid of the student the statements of those who have studied the subject so that he may know the value of mathematics. The only other procedure would be to require some mathematics of all high school students.

Mathematics teaches clear and effective thinking. All real thinking is problem solving. The process as outlined by Prof. E. L. Thorndike, the great psychologist, is as follows:

"1. We must have a clear statement of the problem.

"2. We must select enough of representative individual facts.

"3. These facts must be arranged in such a way as to make the conclusion obvious or necessary.

"4. We must verify the conclusion thus reached by what we already know.

"5. We must apply the conclusion to new situations."

Mathematics gives training in this process. This is the sort of thinking the successful person does many times in a single day. The experience of centuries has shown that the power of logical thinking is best acquired in youth, from mathematics skillfully taught.

The Mathematics Section of the Inland Empire Teachers Association meets in Spokane March 30–April 2, 1921. W. C. Eells of Whitman College is Chairman, and Arthur R. Jerbert of Seattle is Secretary. Professor C. A. Isaacs of the State College at Pullman, Miss Kate Bell of the Lewis and Clark High School in Spokane, Russel L. C. Butsch of Sprague, and Mr. Edwin Twitmyer, State High School Inspector, will discuss the National Committee Report on College Entrance Requirements in Mathematics.

The National Council of Teachers of Mathematics will be discussed by Miss Erma Wylder of the Almira High School, Professor William R. Davis of Whitman College, and by Miss Gertrude Kaye of North Central High School Spokane. (By W. E. Eells.)

The mathematics teachers of Chicago had a rare treat recently when Prof. Louis Karpinski, of Michigan University, spent two days in the city.

During this time Dr. Karpinski visited several colleges and at two o'clock on Friday gave a lecture to about five hundred of the students and faculty of Crane Junior College. The application of mathematics to real life was the main theme of this talk.

At six o'clock, February eighteenth, the Chicago Mathematics Club entertained at dinner in honor of the distinguished guest. About one hundred twenty teachers of mathematics were present.

Following the dinner, Dr. Karpinski gave an illustrated lecture on the History and Development of Algebra, showing slides representing pages from algebras of all time from the Rhind Papyrus of 1600 B. C. to the *first* American mathematics text printed in Mexico in the 16th century,

The lecture was most enjoyable and new ideas regarding the helpfulness of mathematical history in the teaching of algebra were gained by all who heard it.

It is hoped that other events of the same sort will take place during the year. (Zoe Ferguson, Crane Technical High School, Chicago.)

# BOOK REVIEWS.

**Elementary Algebra.** By J. L. NEUFELD. P. Blakiston's Son & Co., Philadelphia. Pp. 383.

Teachers who believe that elementary texts in algebra should be more scholarly and more comprehensive will receive the Neufeld treatise with a great deal of interest. In range of topics and in intensity of treatment this text is comparable to those which appeared twenty or more years ago. The author "does not agree with so-called modern educators who think that too much space is given to the subject of *factoring*. Most failures in algebra can be directly traced to ignorance of *factoring* and the author has therefore given this topic the space its importance demands."

The reader will be interested in the author's selection of content. The theory of negative and fractional exponents, cube root of compound expressions, imaginaries, progressions, inequalities, the binomial theorem with any exponent, permutations and combinations, and logarithms, are among the topics which particularly attract one's attention in examining the text.

It will be interesting to compare the Neufeld book with the Recommendations of the National Committee on Mathematical Requirements.

**Vocational Arithmetic for Girls.** By NETTIE STEWART DAVIS. Bruce Publishing Co., Milwaukee, Wis. Pp. 138.

The author has here embodied her experience as a trade-school teacher into a practical text-book. She has for several years employed a series of lessons prepared by herself which were correlated to the vocational studies engaged in by the girl students.

The book is divided into four parts. Part I. deals with fundamentals in arithmetic, cost of garments, trade problems such as relate to tucking, hemming, ruffling, belt and smocking.

Part II. treats of decimal fractions, the cost of materials, and domestic science problems. Part III. covers percentage, bias and straight bands, pleating problems, carpeting and review problems. Part IV. goes into the problems of the home involving the use of arithmetic.

**Junior High-School Mathematics.** Book III. By JOHN C. STONE. Benjamin Sanborn Co., Chicago. Pp. —.

This third book of the Stone Junior High-School series deserves very careful examination. The content is selected from algebra, intuitive geometry, arithmetic, trigonometry and statistics. There is a sanity of balance and treatment of these topics which will commend the book to all types of schools. It follows, of course, the two preceding books of the same series.

But it is also entitled to consideration as a ninth-grade text in a four-year high school. This volume will be thought of as a modern progressive text.

**Plane Geometry.** By HAWKES, LUBY, AND TOUTON. Ginn and Co., Boston.

Should a geometry be considered from the standpoint that it is to be used somewhat in the ninth grade and finished in the tenth? Or that it is to be used wholly in the tenth? Or wholly in the eleventh? Etc. Texts to be used at these times will be different, for the children differ in capacity. The situation has two conflicting variable factors (1) the time of exposure, and (2) a concept of the subject. When either is preconceived or otherwise determined, a solution of the other can then be sought. It can truly be said that this text is a distinct contribution to the second factor. There is very little discussion *about* geometry, especially in the early portion. Seven pages brings one to the first theorem. Thereafter for a while there is a string of propositions with an almost linear dependence. There is no exercise material in the very early pages. The first exercises are numerical. The proportion of original material increases as the subject develops. The original material is often refreshingly new, interesting, and valuable. This is a quite different conception of the method of developing geometrical ability than that of many texts with their multitude of early exercises. Much information is intended to be developed by "Queries" which ordinarily would be given as corollaries, exercises, etc. This will reduce the time burden and contribute to developing power thoughtfully rather than mechanically. There is a definite attempt to clear up the present unsatisfactory treatment of number. *E.g.*, a sample theorem reads, "An inscribed angle is measured by one half the number of degrees in its intercepted arc." This systematic definite treatment of number is followed consistently throughout. The treatment of loci is distinctive. The authors have followed a development method rather than the usual formal one. The second part of a locus demonstration is treated, proving that a point satisfying the conditions is on the locus rather than proving that those not on the line do not satisfy the conditions. The following have received more extended treatment than is usual (1) the 30-60-90 and 45-45-90 triangles, (2) regular polygons, (3) measurement, (4) pi. All of these are to be commended as given. The figures are excellent; in constructions the arcs, etc., are retained; in theorems on the general polygon the closed polygon is not used. Throughout the book there is an obvious intention not to interfere with a teacher's plan of teaching by giving detailed instructions on organization of methods, summaries, etc. This large measure of freedom in using the book ought to commend it to teachers who can teach plane geometry. In general, the book presents a definite plan of developing geometric power by interesting material with teaching freedom. H. F. H.